6 Expanding I

Brackets are an important part of algebra statements. To multiply out brackets **(expanding)**, multiply each part inside the bracket by the number outside the bracket.

E.g. $2(x + 9) = 2x + 18$ because $2x$ and 18 are different terms, we stop here.

Terms: $(2 \times x)$ (2×9)

Try These

1. $3(x + 3) = 3x +$ _____
2. $4(y - 3) =$ $- 12$ _____
3. $x(4 + y) =$ _____
4. $9(x - 2) =$ _____
5. $7(x - 6) =$ _____
6. $3(x - 7) =$ _____
7. $8(x + 2) =$ _____
8. $x(x + 3) =$ _____
9. $y(x - 5) =$ _____
10. $3(2x + 5) =$ _____

Test Yourself

What is big and red like Santa Claus?

Answer the question by matching the answers with the letter. The first one is done for you.

K $10(x - 2y) = 10x - 20y$

H $-3y(x + 1) =$ _____

R $3(x - y) =$ _____

B $-x(x + 1) =$ _____

T $-3y(x - 2y) =$ _____

A $y(6x + 5) =$ _____

Y $3(2x + 3y) =$ _____

O $-y(x - 3) =$ _____

E $2xy(3x - 4) =$ _____

U $8(2x - y) =$ _____

P $4(x^2 - 2) =$ _____

W $3x(x - 5) =$ _____

$6xy + 5y$ $4x^2 - 8$ $-yx + 3y$ $-3yx - 3y$ $16x - 8y$ $-3yx + 6y^2$ $16x - 8y$ K $10x - 20y$ $6xy + 5y$ $3x^2 - 15x$ $5y + 6xy$

$6y^2 - 3yx$ $3x - 3y$ $6x^2y - 8xy$ $6x^2y - 8xy$

7 Expanding II

To multiply two expressions in brackets, we need to multiply both terms in the first bracket with both terms in the second bracket. The first example shows how this works for a number, the second shows how for expressions in brackets, using numbers and letters. Think of the multiplication as a number.

E.g. (i)
$$\begin{array}{r} 42 \\ \times\ 3 \\ \hline 126 \end{array}$$

(ii)
$$\begin{array}{r} (x+4) \\ \times\ (x+3) \\ \hline 3x + 12 \\ x^2 + 4x \\ \hline x^2 + 7x + 12 \end{array}$$

Keep like terms underneath each other.

as $(x + 4)(x + 3) = x^2 + 3x + 4x + 12$
$= x^2 + 7x + 12$

With practice you will multiply 2 brackets quickly and accurately.

Try These

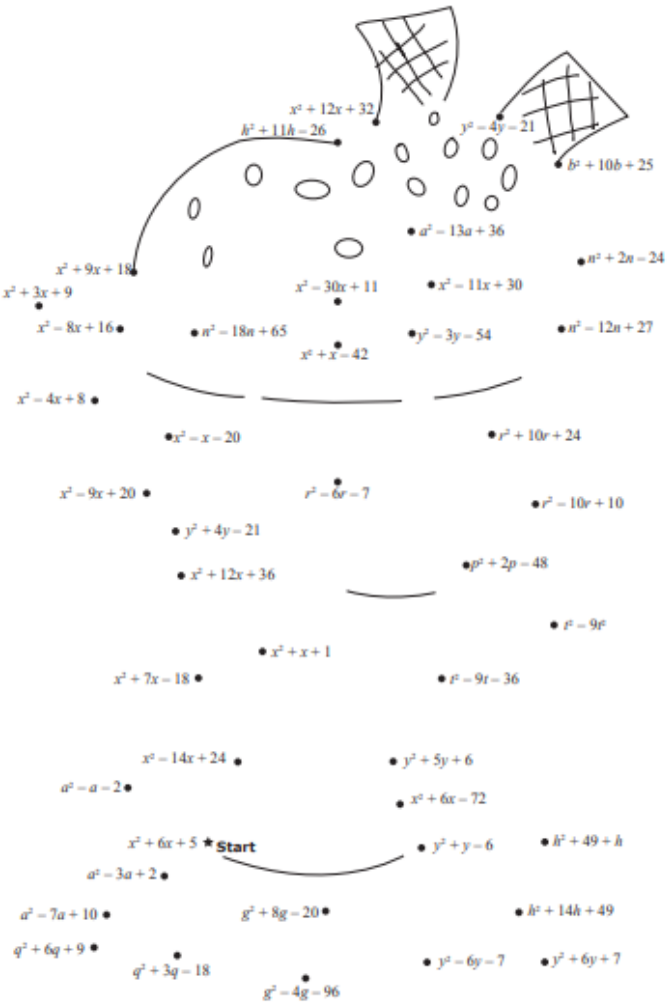
1. $(x + 3)(x + 5) = x^2 + 8x + 15$
2. $(x + 6)(x + 3) =$ _____
3. $(x + 1)(x + 2) =$ _____
4. $(x + 5)(x + 4) =$ _____
5. $(x + 9)(x + 2) =$ _____
6. $(x - 5)(x - 3) = x^2 - 8x + 15$
7. $(x - 2)(x - 12) =$ _____
8. $(x - 3)(x - 11) =$ _____
9. $(x - 4)(x - 2) =$ _____
10. $(x - 5)(x - 7) =$ _____
11. $(x - 5)(x + 4) = x^2 - 1x - 20$
12. $(x + 9)(x - 2) =$ _____
13. $(x - 2)(x + 12) =$ _____
14. $(x + 10)(x - 3) =$ _____
15. $(x + 3)(x - 11) =$ _____
16. $(x + 6)(x - 2) =$ _____
17. $(x + 3)(x - 1) =$ _____
18. $(x + 4)(x - 1) =$ _____
19. $(x + 7)(x - 3) =$ _____
20. $(x - 8)(x + 3) =$ _____

Test Yourself

Expand these expressions.

1. $(x + 5)(x + 1)$ _____
2. $(a - 2)(a - 1)$ _____
3. $(a - 5)(a - 2)$ _____
4. $(q + 6)(q - 3)$ _____
5. $(g - 12)(g + 8)$ _____
6. $(y - 7)(y + 1)$ _____
7. $(h + 7)(h + 7)$ _____
8. $(y + 3)(y - 2)$ _____
9. $(x + 12)(x - 6)$ _____
10. $(y + 3)(y + 2)$ _____
11. $(t - 12)(t + 3)$ _____
12. $(p + 8)(p - 6)$ _____
13. $(r + 6)(r + 4)$ _____
14. $(n - 9)(n - 3)$ _____
15. $(n + 6)(n - 4)$ _____
16. $(b + 5)(b + 5)$ _____
17. $(y - 7)(y + 3)$ _____
18. $(x + 8)(x + 4)$ _____
19. $(h + 13)(h - 2)$ _____
20. $(a - 9)(a - 4)$ _____
21. $(x - 6)(x - 5)$ _____
22. $(y - 9)(y + 6)$ _____
23. $(x + 7)(x - 6)$ _____
24. $(n - 13)(n - 5)$ _____
25. $(x + 6)(x + 3)$ _____
26. $(x - 4)(x - 4)$ _____
27. $(x - 5)(x + 4)$ _____
28. $(y + 7)(y - 3)$ _____
29. $(x + 6)(x + 6)$ _____
30. $(x + 9)(x - 2)$ _____
31. $(x - 2)(x - 12)$ _____

Whew! You made it!



8 Factorising I

Factorising is the reverse process of expanding. We use both processes equally and once expanding is mastered the 'undoing' becomes clear. Always look for the highest common factor.

E.g. i. $4x + 4y = 4(x + y)$

ii. $9x^2 - 18x = 9x(x - 2)$

By multiplying out we get back to the original.

Try These

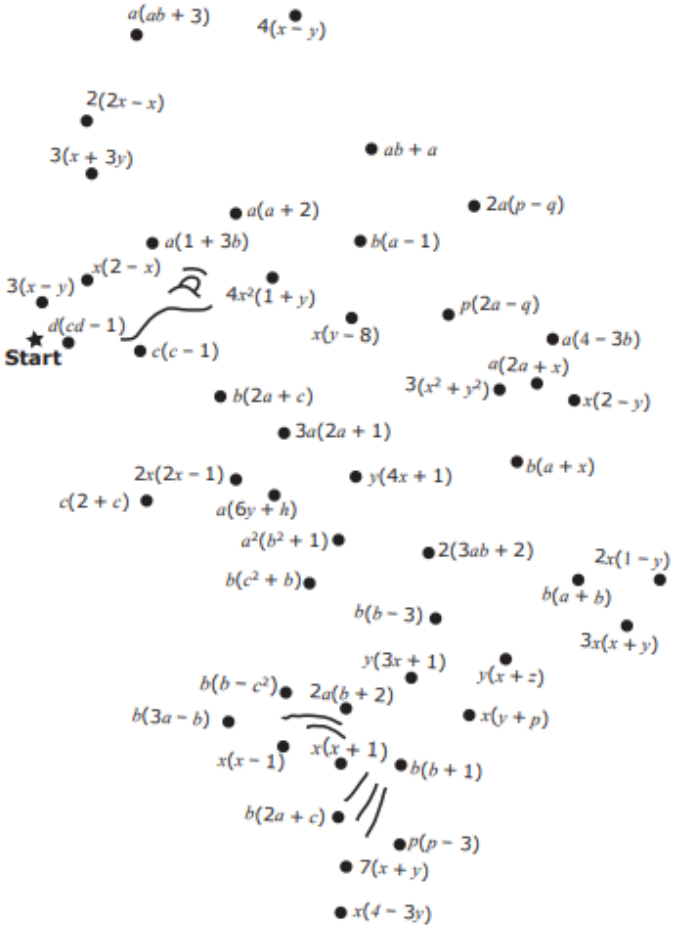
Factorise.

1. $4x + xy = x(\_\_\_\_\_ + \_\_\_\_\_)$
2. $7x - 7y =$ _____
3. $y^2 + 5y = y(\_\_\_\_\_)$
4. $13x - 13q =$ _____
5. $ab + a^2 = a(\_\_\_\_\_ + a)$
6. $10x - 10z =$ _____
7. $a^2 + 2a =$ _____
8. $a^2 - 9a =$ _____
9. $x^2 - 9x =$ _____
10. $2x^2 - 2y =$ _____

Test Yourself

Factorise each equation, find the answer amongst the dots below. Join that dot to the dot of the previous answer. Begin at the star.

1. $3x - 3y$ _____
2. $2x - x^2$ _____
3. $a + 3ab$ _____
4. $a^2 + 2a$ _____
5. $ab - b$ _____
6. $2ap - qp$ _____
7. $4a - 3ba$ _____
8. $2x - yx$ _____
9. $2a^2 + ax$ _____
10. $3x^2 + 3y^2$ _____
11. $ab + bx$ _____
12. $xy + yz$ _____
13. $b^2 + b$ _____
14. $p^2 - 3p$ _____
15. $4x - 3yx$ _____
16. $7x + 7y$ _____
17. $2ab + bc$ _____
18. $x^2 + x$ _____
19. $x^2 - x$ _____
20. $3ab - b^2$ _____
21. $b^2 - bc^2$ _____
22. $2ab + 4a$ _____
23. $3xy + y$ _____
24. $b^2 - 3b$ _____
25. $6ab + 4$ _____
26. $4xy + y$ _____
27. $6a^2 + 3a$ _____
28. $6ay + ah$ _____
29. $4x^2 - 2x$ _____
30. $2ab + cb$ _____
31. $c^2 - c$ _____
32. $cd^2 - d$ _____



9 Factorising II

To factorise a quadratic three-term expression (also known as a *trinomial*) look at the end number and think of two factors which will multiply to the end number. The same two factors must add to the middle number. (You need to know your basic facts!)

E.g. (i) $x^2 + 9x + 20$
$= (x + 5)(x + 4)$

What about 10 x 2 ✗
What about 5 x 4 ✓

E.g. (ii) $x^2 - 1x - 30$
$= (x - 6)(x + 5)$

What about 10 x 3 ✗ because $10 + 3 \neq 1$; $10 - 3 \neq 1$
What about 6 x 5 ✓ because $-6 + 5 = -1$

Note: factors of x^2 are x, x.
No other combination will do.

Order of the brackets doesn't matter, but the sign is crucial.

Try These

1. $x^2 - 11x + 24 = (\ x\ )(\ x - 8\ )$
2. $x^2 + 12x + 36 = (\ x + 6\ )(\ \ )$
3. $y^2 - 9x + 18 = (\ y - 6\ )(\ \ )$
4. $y^2 - 12y + 27 = (\ \ )(\ \ )$
5. $b^2 + 16b + 63 = (\ \ )(\ \ )$
6. $g^2 - 15g + 54 = (\ g - 9\ )(\ \ )$
7. $y^2 + 6y + 9 = (\ \ )(\ \ )$
8. $h^2 + 8h + 12 = (\ \ )(\ \ )$
9. $x^2 + 8x + 16 = (\ \ )(\ \ )$
10. $r^2 - 20r + 96 = (\ \ )(\ r - 12\ )$
11. $y^2 - 16y + 48 = (\ y - 12\ )(\ \ )$
12. $a^2 + 14a + 33 = (\ \ )(\ \ )$
13. $x^2 + x - 12 = (\ \ )(\ \ )$
14. $y^2 + 2y - 48 = (\ \ )(\ \ )$
15. $x^2 + 3x - 18 = (\ \ )(\ x + 6\ )$
16. $x^2 - 4x - 12 = (\ x - 6\ )(\ \ )$
17. $x^2 - x - 20 = (\ \ )(\ \ )$
18. $y^2 - 3y - 40 = (\ y - 8\ )(\ \ )$
19. $x^2 - 7x - 8 = (\ \ )(\ \ )$
20. $x^2 - x - 42 = (\ \ )(\ \ )$

10 Factorising III

There are two special types of factorising which can be useful to recognise.

i. **Perfect Squares**

$x^2 + 12x + 36 = (x + 6)(x + 6)$
$= (x + 6)^2$

Can you see a pattern here?
(double 6, square 6)

ii. **Difference of 2 squares**

$x^2 - 9 = (x + 3)(x - 3)$

Note that the middle term disappears because $+3x - 3x = 0$

Try These

1. Factorise these perfect squares.

a. $x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2$
b. $x^2 - 8x + 16 = (\ x - 4\ )(\ \ ) =$
c. $x^2 - 10x + 25 =$
d. $y^2 + 18y + 81 = (y + 9)(\ \ ) = (\ \ )^2$
e. $x^2 - 20x + 100 =$
f. $x^2 + 30x + 225 =$
g. $y^2 + 3y + 2.25 =$
h. $x^2 - 14x + 49 = (\ x - 7\ )$
i. $y^2 - 22y + 121 =$
j. $x^2 - 2xy + y^2 =$

2. Factorise these 'difference of 2 squares'.

a. $x^2 - 81 = (\ x + 9\ )(\ x - 9\ )$
b. $x^2 - y^2 = (\ x - y\ )(\ \ )$
c. $x^2 - 16 = (\ x - 4\ )(\ \ )$
d. $y^2 - 1 =$
e. $x^2 - 100 =$
f. $y^2 - 49 =$
g. $q^2 - 4 =$
h. $y^2 - 36 =$
i. $x^2 - 4y^2 =$
j. $4x^2 - 1 =$